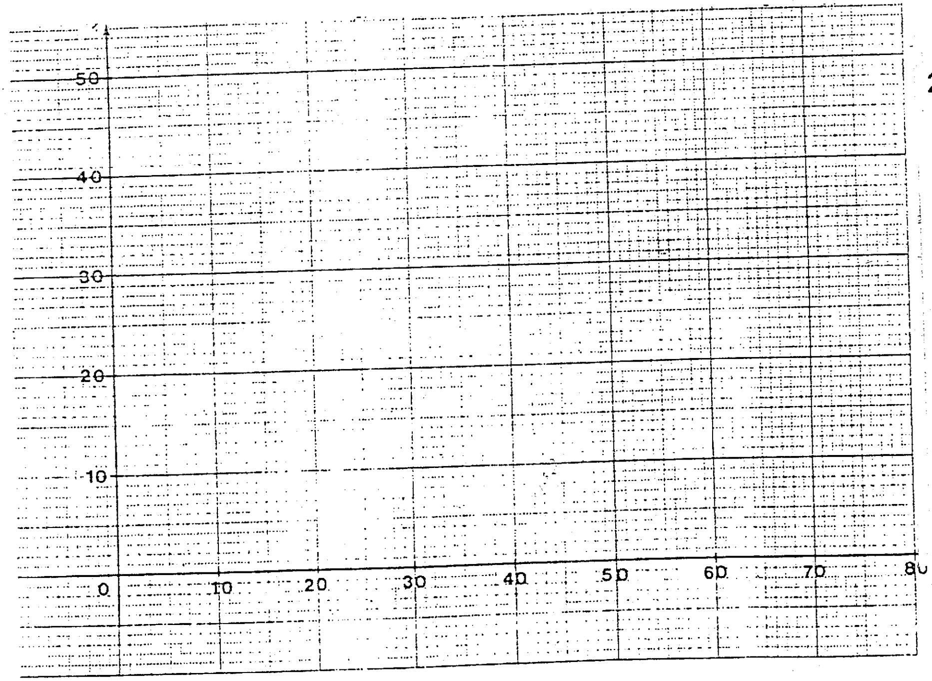
Inequalities and Linear Programming
HKCEE Problems

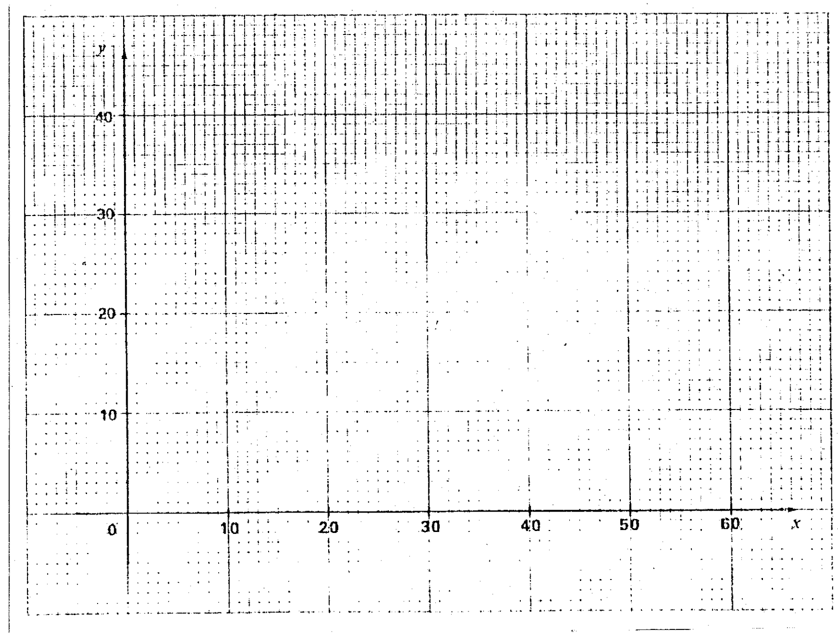
(80) An airline company has a small passenger plane with a luggage capacity of 720 kg, and a floor area of 60 m^2 for installing passenger seats. An economy-class seat takes up 1 m^2 of floor area while a first-class seat takes up 1.5 m^2. The company requires that the number of first-class seats should not exceed the number of economy-class seats. An economy-class passenger cannot carry more than 10 kg of luggage while a first-class passenger cannot carry more than 30 kg of luggage.

The profit from selling a first-class ticket is double that from selling an economy-class ticket. If all tickets are sold out in every flight, find graphically how many economy-class seats and how many first-class seats should be installed to give the company the maximum profit.

(10 marks)

(Let x be the number of economy-class seats installed,
y be the number of first-class seats installed.)





2(81) An association plans to build a hostel with x single rooms and y double rooms satisfying the following conditions :

(1) The hostel will accommodate at least 48 persons.
(2) Each single room will occupy an area of 10 m^2, each double room will occupy an area of 15 m^2 and the total available floor area for the rooms is 450 m^2.
(3) The number of double rooms should not exceed the number of single rooms.

If the profits on a single room and a double room are \$300 and \$400 per month respectively, find graphically the values of x and y so that the total profit will be a maximum.

(12 marks)

3(82) Solve $2x^2 - x < 36$. (5 marks)

83) (a) On the graph paper provided below, draw the following straight lines:

$$y = 2x,$$
$$x + y = 30,$$
$$2x + 3y = 120.$$

(3 marks)

(b) On the same graph paper, shade the region that satisfies all the following inequalities:

$$\begin{cases} y \geqslant 0, \\ y \leqslant 2x, \\ x + y \geqslant 30, \\ 2x + 3y \leqslant 120. \end{cases}$$

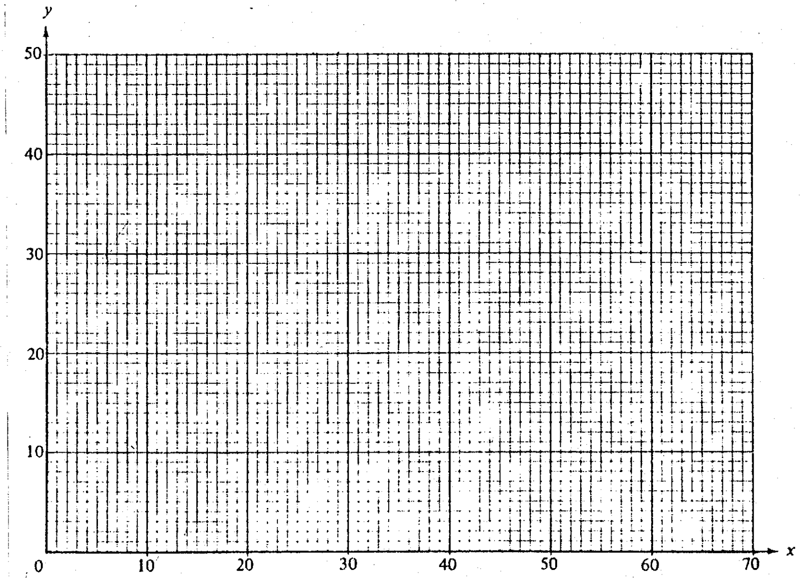
(3 marks)

(c) It is given that $P = 3x + 2y$.
Under the constraints given by the inequalities in (b),

(i) find the maximum and minimum values of P, and

(ii) find the maximum and minimum values of P if there is the additional constraint $x \leqslant 45$.

(6 marks)



5(84)

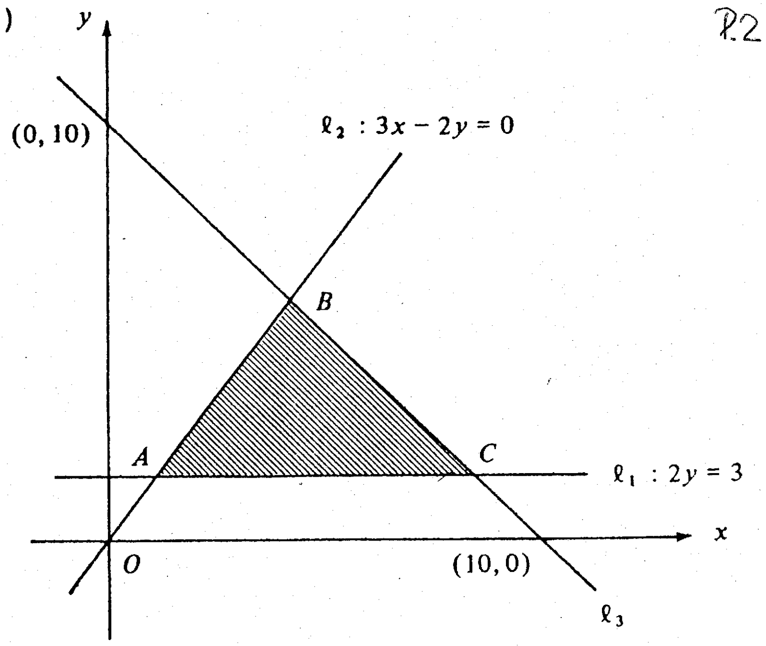


Figure 2

In Figure 2, $\ell_1 : 2y = 3$,

$\ell_2 : 3x - 2y = 0$.

The line ℓ_3 passes through $(0, 10)$ and $(10, 0)$.

(a) Find the equation of ℓ_3.

(2 marks)

(b) Find the coordinates of the points A, B and C.

(3 marks)

(c) In Figure 2, the shaded region, including the boundary, is determined by three inequalities. Write down these inequalities.

(3 marks)

(d) (x, y) is any point in the shaded region, including the boundary, and $P = x + 2y - 5$. Find the maximum and minimum values of P

(4 marks)

(86) (a) (i) On the graph paper provided, draw the following straight lines :

$$x + y = 40,$$

$$x + 3y = 60,$$

$$7x + 2y = 140.$$

(ii) On the same graph paper, shade the region that satisfies all the following constraints :

$$x \geqslant 0,$$

$$y \geqslant 0,$$

$$x + y \geqslant 40,$$

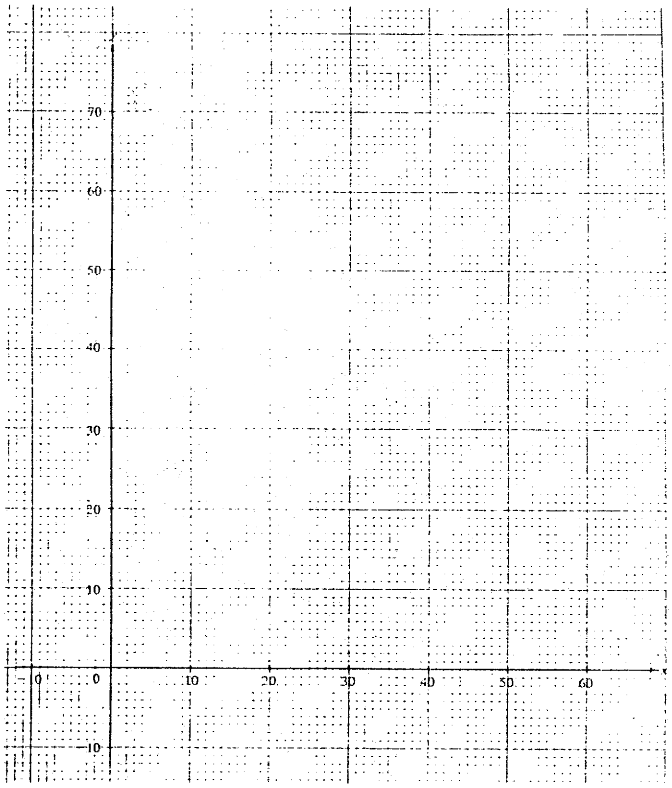
$$x + 3y \geqslant 60,$$

$$7x + 2y \geqslant 140.$$

(6 marks)

(b) A company has two workshops A and B. Workshop A produces 1 cabinet, 1 table and 7 chairs each day; Workshop B produces 1 cabinet, 3 tables and 2 chairs each day. The company gets an order for 40 cabinets, 60 tables and 140 chairs. The expenditures to operate Workshop A and Workshop B are respectively \$1000 and \$2000 each day. Use the result of (a)(ii) to find the number of days each workshop should operate to meet the order if the total expenditure in operating the workshops is to be kept to a minimum. (Denote the number of days that Workshops A and B should operate by x and y respectively.)

(6 marks)



7(87) A factory produces three products A, B and C from two materials M and N.

Each tonne of M produces 4000 pieces of A, 20 000 pieces of B and 6000 pieces of C.

Each tonne of N produces 6000 pieces of A, 5000 pieces of B and 3000 pieces of C.

The factory has received an order for 24 000 pieces of A, 60 000 pieces of B and 24 000 pieces of C. The costs of M and N are respectively \$4000 and \$3000 per tonne. By following the steps below, determine the least cost of the materials used so as to meet the order.

(a) Suppose x tonnes of M and y tonnes of N were used. By considering the requirement of A, B and C of the order, five constraints could be obtained. Three of them are:

$$x \geqslant 0,$$
$$y \geqslant 0,$$
$$4000x + 6000y \geqslant 24\,000.$$

Write down the other two constraints on x and y.

(2 marks)

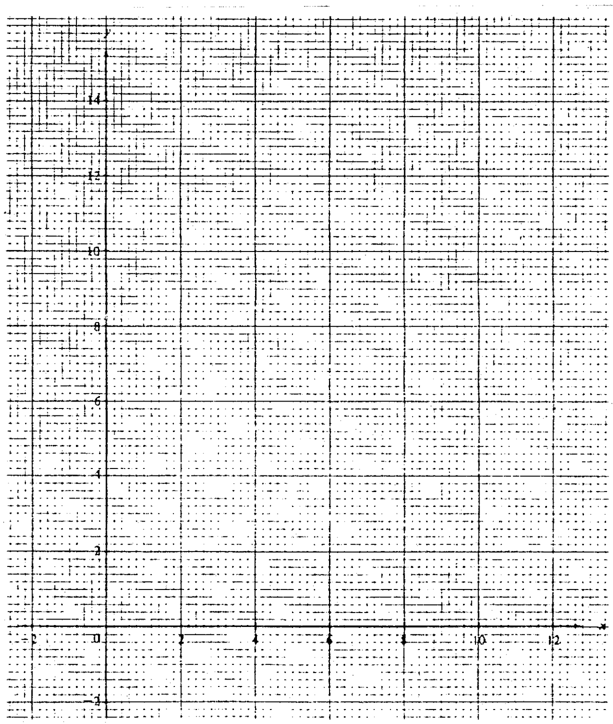
(b) On the graph paper provided, draw and shade the region which satisfies the five constraints in (a).

(6 marks)

(c) Express the cost of materials in terms of x and y.

Hence use the graph in (b) to find the least cost of materials used to meet the order.

(4 marks)



8(88) In Figure 5, L_1 is the line $x = 3$ and L_2 is the line $y = 4$. L_3 is the line passing through the points (3, 0) and (0, 4).

(a) Find the equation of L_3 in the form $ax + by = c$, where a, b and c are integers.

(2 marks)

(b) Write down the three constraints which determine the shaded region, including the boundary.

(3 marks)

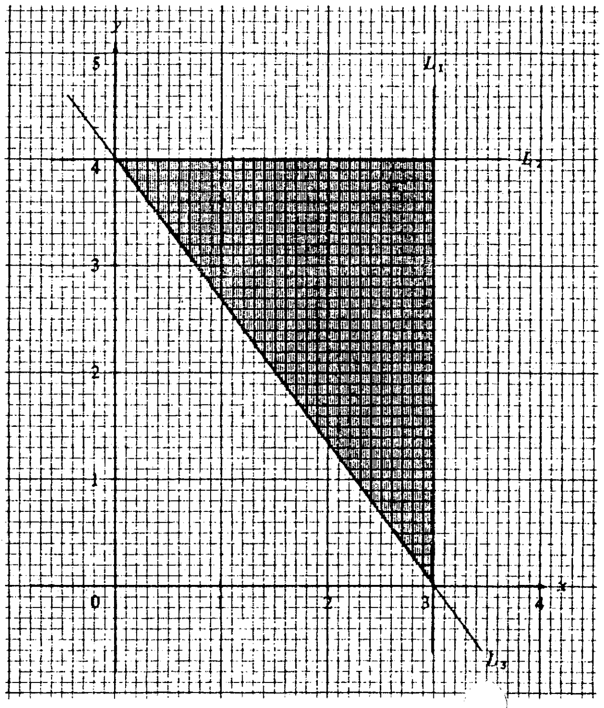
(c) Let $P = x + 4y$. If (x, y) is any point satisfying all the constraints in (b), find the greatest and the least values of P.

(4 marks)

(d) If one more constraint $2x - 3y + 3 \leqslant 0$ is added, shade in Figure 5 the new region satisfying all the four constraints.

For any point (x, y) lying in the new region, find the least value of P defined in (c).

(3 marks)



9(88) Solve the inequality $2x^2 \geqslant 5x$

(5 marks)

10(89) Consider $x + 1 > \frac{1}{5}(3x + 2)$.

(a) Solve the inequality.

(b) In addition, if $-4 \leqslant x \leqslant 4$, find the range of x.

(4 marks)

11(89) **(a) In Figure 6, draw and shade the region that satisfies the following inequalities:**

$$\begin{cases} y \geqslant 20 \\ 2x - y \geqslant 40 \\ x + y \leqslant 100 \end{cases}$$

(4 marks)

(b) The vitamin content and the cost of three types of food X, Y and Z are shown in the following table:

	Food X	Food Y	Food Z
Vitamin A (units/kg)	400	600	400
Vitamin B (units/kg)	800	200	400
Cost (dollars/kg)	6	5	4

A man wants to produce 100 kg of a mixture by mixing these three types of food. Let the amount of food X, food Y and food Z used be x, y and z kilograms respectively.

(i) Express z in terms of x and y.

(ii) Express the cost of the mixture in terms of x and y.

(iii) Suppose the mixture must contain at least 44 000 units of vitamin A and 48 000 units of vitamin B. Show that

$$\begin{cases} y \geqslant 20 \\ 2x - y \geqslant 40 \\ x + y \leqslant 100 \end{cases}$$

(iv) Using the result in (a), determine the values of x, y and z so that the cost is the least.

(8 marks)

12(90) (a) Solve the following inequalities:

(i) $6x + 1 \geqslant 2x - 3$,

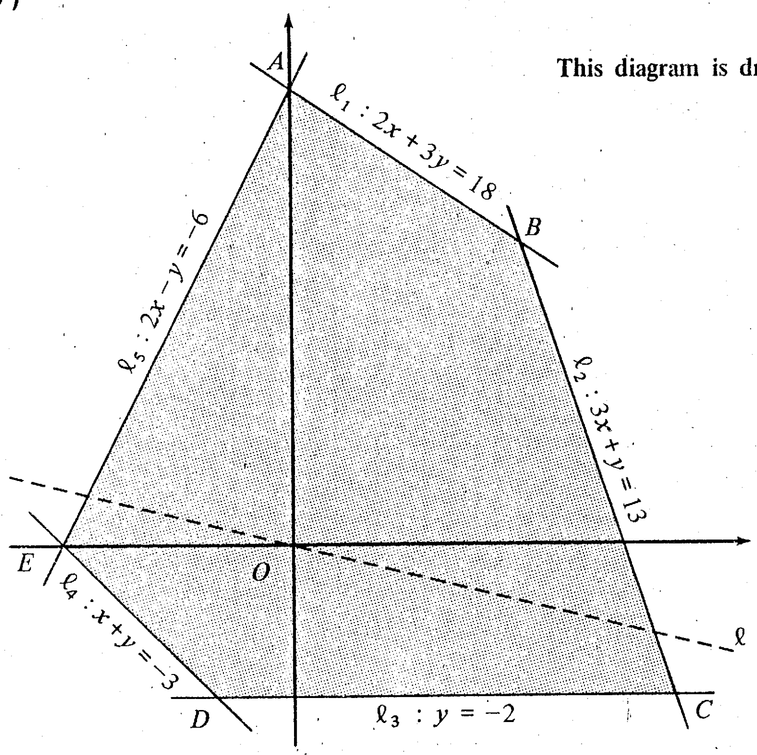
(ii) $(2 - x)(x + 3) > 0$.

(b) Using (a), find the values of x which satisfy

both $6x + 1 \geqslant 2x - 3$ and $(2 - x)(x + 3) > 0$.

(6 marks)

13(90)



This diagram is drawn to scale.

Figure 1

In Figure 1, the shaded region $ABCDE$ is bounded by the five given lines ℓ_1 , ℓ_2 , ℓ_3 , ℓ_4 and ℓ_5 . The line $\ell : x + 4y = 0$ passes through the origin O .

Let $P = x + 4y - 2$, where (x , y) is any point in the shaded region including the boundary. Find the greatest and the least values of P .

(6 marks)

14(91) In Figure 3, L_1 is the line $x = 4$, L_2 is the line passing through the point (0, 2) with slope 1, and L_3 is the line passing through the points (5, 0) and (0, 5).

(a) Find the equations of L_2 and L_3 .

(3 marks)

(b) Write down the three inequalities which determine the shaded region, including the boundary.

(3 marks)

(c) Suppose $P = x + 2y - 3$ and (x, y) is any point satisfying all the inequalities in (b).

(i) Find the point (x, y) at which P is a minimum. What is this minimum value of P ?

(ii) If $P \geq 7$, by adding a suitable straight line to Figure 3, find the range of possible values of x .

(6 marks)

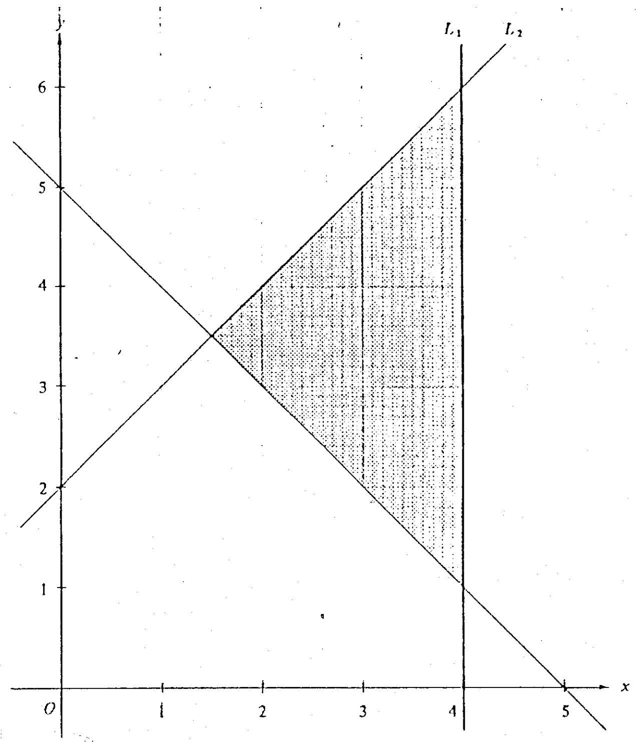


Figure 3

HKCEE M.C.
INEQUALITIES AND LINEAR PROGRAMMING

1. $2x - 3a - 4 > 3x + 5a + 6$ is equivalent to
(83) A. $x > -8a - 10$ B. $x > 2a - 10$ C. $x < -8a - 10$
D. $x < (2a+2)/5$ E. $x > (2a+2)/5$

2. $12 - x - x^2 < 0$ is equivalent to
(83) A. $x < -4$ B. $x > 3$ C. $-4 < x < 3$
D. $x < -3$ or $x > 4$ E. $x < -4$ or $x > 3$

3. $4x^2 - 9 \geq 0$ is equivalent to
(84) A. $x \geq 3/2$ or $x \geq -3/2$ B. $3/2 \leq x \leq -3/2$
C. $-3/2 \leq x \leq 3/2$ D. $x \geq -3/2$ or $x \leq 3/2$
E. $x \leq -3/2$ or $x \geq 3/2$

4. If a and b are non-zero real numbers and $a > b$, which
(84) of the following must be true ?
(1) $a^2 > b^2$ (2) $1/a > 1/b$ (3) $a^3 > b^3$
A. (2) only B. (3) only C. (1) and (2) only
D. (2) and (3) only E. (1) and (3) only

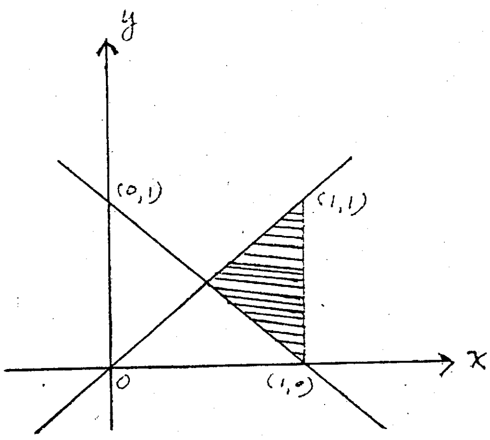
5. Which of the following is the solution of
(85) $(x-1)(x-3) \leq 0$ and $x - 2 \leq 0$?
A. $x \leq 2$ B. $x \leq 3$ C. $2 \leq x \leq 3$ D. $1 \leq x \leq 2$
E. $1 \leq x \leq 3$

6. Which of the following systems of
(85) inequalities determine the shaded region in the figure ?

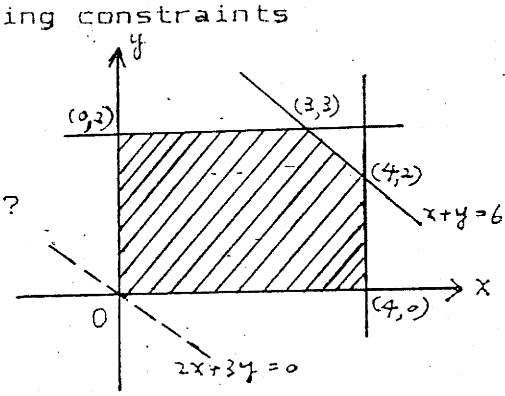
A. $\begin{cases} x \geq 1 \\ x+y \geq 1 \\ x \geq y \end{cases}$ B. $\begin{cases} x \geq 1 \\ x+y \leq 1 \\ x \geq y \end{cases}$

C. $\begin{cases} x \leq 1 \\ x+y \leq 1 \\ x \leq y \end{cases}$ D. $\begin{cases} x \leq 1 \\ x+y \leq 1 \\ x \geq y \end{cases}$

E. $\begin{cases} x \leq 1 \\ x+y \geq 1 \\ x \geq y \end{cases}$



7. Let $p = 2x + 3y$. Under the following constraints
(86) $\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 4 \\ y \leq 3 \\ x+y \leq 6, \end{cases}$
which is the greatest value of p ?
A. 8
B. 14
C. 15
D. 16
E. 17



8. If $a > 0$ and $b < 0$, which of the following is/are
(86) negative ?
(1) $1/a - 1/b$ (2) $a/b + b/a$ (3) $a^2/b - b^2/a$
A. (1) only B. (3) only C. (1) and (2) only
D. (1) and (3) only E. (2) and (3) only

9. (86) If $2 < x < 3$ and $3 < y < 4$, then the range of values of x/y is
A. $1/2 < x/y < 3/4$ B. $1/2 < x/y < 1$ C. $2/3 < x/y < 3/4$
D. $2/3 < x/y < 1$ E. $4/3 < x/y < 3/2$

10. (87) If x and y are integers with $x > y$, which of the following is/are true ?
(1) $x^2 > y^2$ (2) $1/x < 1/y$ (3) $10^x > 10^y$
A. (3) only B. (1) and (2) only C. (1) and (3) only
D. (2) and (3) only E. (1),(2) and (3)

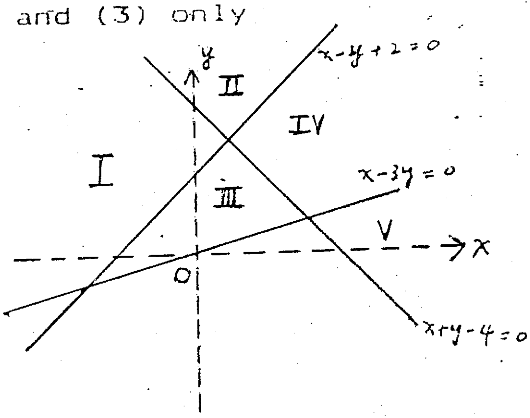
11. (87) Solve the inequality $x\log_{10}0.1 > \log_{10}10$
A. $x > -1$ B. $x > 1$ C. $x > 100$ D. $x < 1$ E. $x < -1$

12. (88) If $\log a > 0$ and $\log b < 0$, which of the following is/are true ?
(1) $\log(a/b) > 0$ (2) $\log b^2 > 0$ (3) $\log(1/a) > 0$
A. (1) only B. (2) only C. (3) only
D. (1) and (2) only E. (2) and (3) only

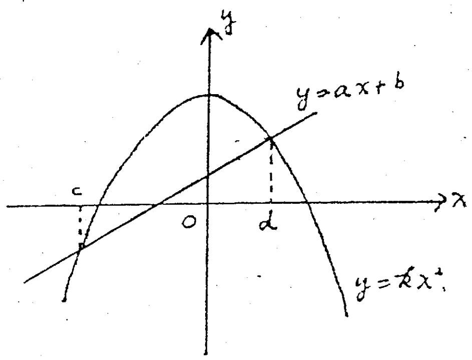
13. (88) In the figure, which region represents the solution to the following inequalities ?

$$\begin{cases} x - 3y \le 0 \\ x - y + 2 \ge 0 \\ x + y - 4 \ge 0 \end{cases}$$

A. I
B. II
C. III
D. IV
E. V



14. (88) In the figure, the line $y=ax+b$ cuts the curve $y=kx^2$ at $x=c$ and $x=d$. Find the range of values of x for which $kx^2 < ax + b$
A. $c < x < d$
B. $c < x < 0$
C. $x < c$ or $x > d$
D. $x < c$
E. $x > d$



ANSWERS
1.C 2.E 3.E 4.B 5.D 6.E 7.C 8.E 9.B 10.A
11.E 12.A 13.D 14.C

Inequalities and Linear Programming

1. $2x-3a-4>3x+5a+6$

$-x>8a+10$

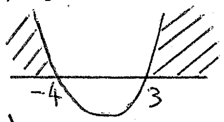
$\therefore x<-8a-10$ (A.)

2. $12-x-x^2<0$

$x^2+x-12>0$

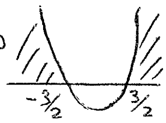
$(x+4)(x-3)>0$

$\therefore x<-4$ or $x>3$. (E.)



3. $4x^2-9\geq 0$

$(2x-3)(2x+3)\geq 0$



$x\leq -\frac{3}{2}$ or $x\geq \frac{3}{2}$. (E.)

4. a, b are non-zero real numbers.

$a>b$.

(1) if b is negative.

$a^2<b^2$

$\therefore$ (1) is not true.

(2). $a>b$

$\frac{1}{a}<\frac{1}{b}$.

$\therefore$ (2) is not true.

(3). if a, b are positive.

$a>b$.

$a^3>b^3$.

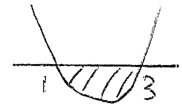
if. a, b are negative

$a>b$

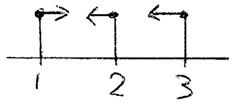
$a^3>b^3$.

$\therefore$ (3) is ture. (B.)

5. $\begin{cases}(x-1)(x-3)\leq 0\\ x-2\leq 0\end{cases}$

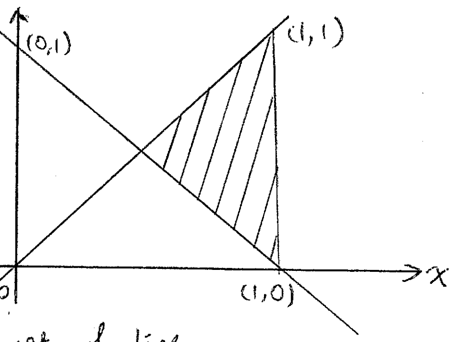


$\begin{cases}1\leq x\leq 3\\ x\leq 2\end{cases}$



$\therefore 1\leq x\leq 2$ (D.)

6.



the eqt. of line.

$(y-0)=(\frac{1-0}{1-0})(x-0)$

$y=x$. ——(1)

$\frac{x}{1}+\frac{y}{1}=1$.

$x+y=1$ ——(2).

$x=1$ ——(3).

Checking the points.

(1). $x=y$.

put. (1,0).

$1>0$

$\therefore x\geq y$.

(2). $x+y=1$.

put (1,1).

$1+1>1$.

$\therefore x+y\geq 1$.

(3). $x=1$.

put. (0,0).

$\therefore 0<1$.

$\therefore x\leq 1$.

$\therefore$ the system is

$\begin{cases}x\geq y\\ x+y\geq 1\\ x\leq 1\end{cases}$ (E.)

7. $P=2x+3y$.

Check the extreme points.

(0,2).

$P=(2)(0)+3(2)$

$=6$.

(4,0)

$P=(2)(4)+3(0)$

$=8$.

(3,3)

$P=(2)(3)+3(3)$

$=15$

(4,2)

$P=(2)(4)+3(2)$

$=14$.

$\therefore P_{max}=15$. (C.)

8. $\begin{cases}a>0\\ b<0\end{cases}$

(1). $\frac{1}{a}>0$, $-\frac{1}{b}>0$.

$\therefore \frac{1}{a}-\frac{1}{b}>0$.

(2) $a>0$, $b<0$.

$\frac{1}{a}>0$, $\frac{1}{b}<0$.

$\therefore a\cdot\frac{1}{b}=\frac{a}{b}<0$

$b\cdot\frac{1}{a}=\frac{b}{a}<0$

$\therefore \frac{a}{b}+\frac{b}{a}<0$.

(3). $a^2>0$, $\frac{1}{b}<0$

$\frac{1}{a}>0$, $b^2>0$.

$a^2\cdot\frac{1}{b}=\frac{a^2}{b}<0$.

$b^2\cdot\frac{1}{a}=\frac{b^2}{a}>0$

$\therefore \frac{a^2}{b}-\frac{b^2}{a}<0$.

(2) and (3) only (E.)

4. $\begin{cases} 2 < x < 3 \\ 3 < y < 4 \end{cases}$

$\frac{1}{3} > \frac{1}{y} > \frac{1}{4}$

$\therefore 2 \cdot \frac{1}{4} < x \cdot \frac{1}{y} < 3 \cdot \frac{1}{3}$

$\frac{1}{2} < \frac{x}{y} < 1$ (B.)

10. $x > y$ x, y are integers.

(1) if $x, y > 0$.

$x^2 > y^2$

if $x, y < 0$.

$x^2 < y^2$.

$\therefore$ (1) is not true.

(2) if. $x > 0$, $y < 0$.

when. $x > y$

$\frac{1}{x} > \frac{1}{y}$.

$\therefore$ (2) is not true.

(3). $10^x > 10^y$

$x \log_{10} 10 > y \log_{10} 10$

$x > y$

$\therefore$ (3) is true only (A.)

$x \log_{10} 0.1 > \log_{10} 10$

$x(-1) > 1$.

$-x > 1$

$x < -1$. (E.)

2. $\log a > 0$

$\log b < 0$.

(1). $\log(a/b)$

$= \log a - \log b$

> 0.

(2) $\log b$

$= 2 \log b$

< 0

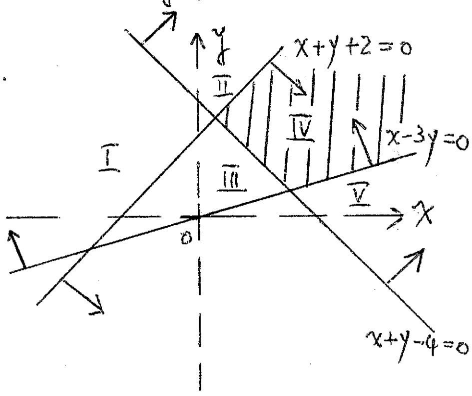
(3) $\log(1/a)$

$= -\log a$

< 0.

$\therefore$ (1) only (A.)

13.



$\begin{cases} x - 3y \le 0 & \text{——①} \\ x - y + 2 \ge 0. & \text{——②} \\ x + y - 4 \ge 0 & \text{——③} \end{cases}$

① Check. (0,1).

$\therefore$ $0 - 3(1) < 0$.

② Check (0,0).

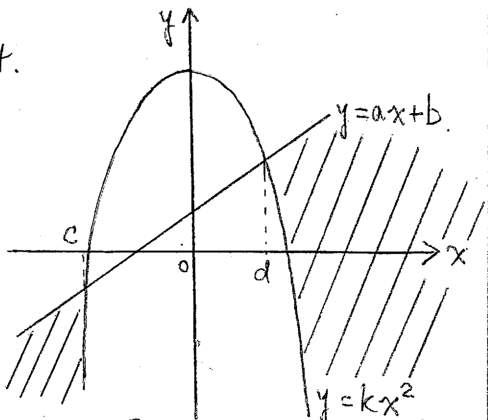
$0 - 0 + 2 > 0$.

③ Check (0,0)

$0 + 0 - 4 < 0$.

$\therefore$ IV (D.)

14.



$\begin{cases} y = kx^2 \\ y = ax + b \end{cases}$

$kx^2 < ax + b$

$\therefore x < c$ or $x > d$

(C.)